Statistical Analysis of Low Frequency Inversion Spectrum Pulsars

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Abstract: Low-frequency turnover pulsars refer to pulsars that exhibit a spectral turnover near 100 MHz. To date, 82 pulsars with low-frequency turnover spectra have been discovered. The formation of this special type of radio spectrum is likely due to more intense free-free absorption in the interstellar medium at low frequencies. In this study, we explore and analyze the dispersion measure, peak frequency, period, and magnetic field strength of these pulsars. We find that the turnover frequencies of low-frequency turnover pulsars mainly range from 0.05 GHz to 0.2 GHz, and the average spectral index is -2.23. The dispersion measures of low-frequency turnover pulsars are mainly distributed between 0 and 50 pc cm^{-3}, and there is no obvious correlation between the dispersion measure and period. However, there is a clear negative correlation between age and magnetic field strength, indicating that the magnetic field strength of low-frequency turnover pulsars will continuously decay with increasing age.

1. Introduction

The first discovery of a pulsar was made by Hewish et al. (1968). Since its discovery, various methods have been used to explore these mysterious celestial bodies, and to date, over 3000 pulsars have been detected. Pulsars are known as "cosmic clocks" and are a type of natural high-precision timer. They not only emit regular pulse signals but also these signals are extremely stable. The radiation spectrum of pulsars has been extensively studied for decades, as it provides valuable insights into the internal structure and radiation mechanisms of these objects. By analyzing the radiation spectrum of radio pulsars, researchers can gain a deeper understanding of their properties and behavior. Therefore, the radiation spectrum remains a highly relevant and widely researched topic in this field. Sieber (1973) was the first to study the radio emission spectra of a large number of pulsars and found that most pulsars can be described by a simple power-law spectrum with a spectral index of $\alpha : S_v = S_0 v^a$, where S_v is the flux density of the pulsar at frequency v, S_0 is the flux density at a frequency of 1 GHz, and α is the spectral index, which is generally a negative value. Sieber (1973) also discovered a special type of pulsar, which exhibits a particular spectral shape known as the turnover spectrum. These pulsars have a peak in their flux density at several hundred megahertz, with the maximum value appearing at around 100 MHz, and the flux density decreases as the frequency increases. Subsequently, Malofeev et al. (1980) and Izvekova et al. (1981) also began to study the low-frequency region below 100 MHz. We coined the term MPS (megahertz-peaked spectra) pulsars for the first time to describe pulsars with this special spectral shape. Jankowski et al. (2018) identified 10 pulsars with turnovers occurring in the range of 50MHz-100MHz out of 441 pulsars surveyed and pointed out that this turnover phenomenon is related to the free-free absorption (FFA) occurring in the interstellar medium (such as stellar wind clouds and supernova remnants) surrounding pulsars. At lower frequencies, this absorption becomes more intense, which leads to the turnover in the pulsar spectra at lower frequencies. Studies on the low-frequency turnover phenomenon in pulsars have been conducted by Noutsos et al. (2015), Bilous et al. (2016), Murphy et al. (2017), and Bondonneau et al. (2020). These studies have indicated that the interstellar medium can affect the measurement of flux density and have again suggested that the low-frequency turnover may be related to free-free absorption by the interstellar

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medium. However, some conclusions suggest that the low-frequency turnover may not only be related to the interstellar medium but also to the absorption of radiation by plasma in the pulsar magnetosphere or a decrease in radiation efficiency. Observations at high frequencies are typically less affected by the interstellar medium, while observations at low frequencies are more strongly impacted by it. Therefore, low-frequency observations of pulsars are more advantageous for studying the composition of the interstellar medium surrounding them. One of the objectives of studying the low-frequency turnover spectra of pulsars is to collect more samples of megahertz-peaked spectrum (MPS) pulsars and analyze the relationship between the turnover and other parameters, explore the radiation mechanism behind the low-frequency turnover phenomenon, and understand the composition of the interstellar medium surrounding pulsars. From a macro perspective, studying the low-frequency turnover phenomenon of pulsars is more conducive to detecting low-frequency gravitational waves, and thereby gaining insights into the formation structure of black holes and the universe. In addition to some typical MPS (megahertz-peaked spectra) pulsars showing low-frequency turnovers, it has been found that some millisecond pulsars (MSPs) also exhibit this phenomenon. Kuzmin and Losovsky (2001) observed 30 MSPs and found that only J1012+5307 showed a low-frequency turnover. They suggested that the lack of low-frequency turnovers in MSPs is due to their different magnetic field structure compared to typical pulsars. Currently, the low-frequency turnover phenomenon in millisecond pulsars (MSPs) is still a hot research topic. For example, Kuniyoshi et al. (2015) identified the spectral indices of three new MSPs and determined that six MSPs may experience low-frequency turnover. Lee et al. (2022) observed 22 pulsars, eight of which showed low-frequency turnover, including the MSP J0437-4715, which exhibited turnover around 285 MHz. Due to their stable periods, MSPs are very suitable for low-frequency gravitational wave detection. Scientists hope to use millisecond pulsars as precise cosmic clocks to detect the displacement of Earth caused by gravitational waves. The study of low-frequency turnovers in millisecond pulsars plays an important role in improving the sensitivity of detecting low-frequency gravitational waves. Moreover, the phenomenon of low-frequency turnovers in pulsars is essential for understanding pulsar evolution and the state of the interstellar medium.

This paper aims to study the spectral turnovers of MPS pulsars, a type of special spectrum that has not been systematically discussed yet. Usually, the free-free absorption model is used to fit such spectra. We selected 82 MPS pulsars from a large sample of pulsars and analyzed and summarized their peak frequency, spectral index, period, period derivative, magnetic field, and other parameters to deepen our understanding of the reasons behind the spectral turnovers of MPS pulsars.

2. Radiation Mechanism of MPS Pulsars

During collisions with ions, electrons not only emit photons and undergo free-free radiation, but they can also absorb photons, causing the transition of free electrons from a lower kinetic energy state to a higher one. This process is the inverse of free-free radiation, known as free-free absorption. Free-free absorption in the interstellar medium affects the flux density of pulsars and causes their radiation spectra to exhibit turnovers. Self-absorption in synchrotron radiation occurs when electrons absorb photons generated by their own synchrotron radiation. Unlike self-absorption in synchrotron radiation, free-free absorption is more significant at low frequencies, and pulsars that experience free-free absorption are typically located in dense environments such as pulsar wind nebulae, supernova remnants, or other high-density regions, where their interaction with the electron density and temperature of the environment is crucial. The notable difference between synchrotron self-absorption and free-free absorption can be seen in the spectral shape of a uniform synchrotron radiation source. At low frequencies, the radiation spectrum affected by free-free absorption drops faster than that affected by synchrotron self-absorption. Lewandowski et al. (2015) found that the impact of free-free absorption on the pulsar radiation spectrum is related to the size of the absorption region, electron density, and electron temperature. Moreover, the strength of free-free absorption is highly sensitive to changes in electron density. High-density and high-temperature environments can cause significant absorption of the pulsar's flux density by the surrounding matter, leading to a reversal of the spectrum's peak frequency. Initially, the free-free absorption model was applied in non-pulsar fields, but further exploration revealed its excellent applicability in studying pulsar reversal spectra. Numerous studies have demonstrated that this model is suitable for describing the reversed spectra of gigahertz-peaked spectra (GPS) pulsars. For instance, Kijak et al. (2017) used this model to fit the reversed spectra of GPS pulsars. Unlike the low-frequency turnover spectra, GPS (gigahertz-peaked spectra) are high-frequency turnover spectra with peak inversion frequencies occurring around 1 GHz. In this paper, we adopt the free-free absorption model to describe the low-frequency turnover phenomenon in MPS (mid-frequency peaked spectrum) pulsars. We use the expression proposed by Kameno et al. (2000) for the formula, which takes the following form:

$$S_v = S_0 v^a \exp(-\tau_f v^{-2.1}) \tag{1}$$

Where v and S_v are the observing frequency (Frequency) and its corresponding flux density (Flux Density) in units of GHz and mJy, respectively. S_0, τ_f, and α are the flux density at 1 GHz, free-free absorption coefficient, and high-frequency spectral index, respectively. The free-free absorption coefficient τ_f is mainly determined by the electron density, temperature, and size of the absorbing medium.

3. Statistical Study of Low Frequency Inversion Spectrum Pulsars

In this paper, 82 pulsars with peak frequencies below 0.45 GHz and exhibiting low-frequency turnovers in their spectra were selected from the literature, as shown in Table 1. The spectral index in the fourth column and the peak frequency in the fifth column of Table 1 were obtained from the references in the eighth column, while other parameters in Table 1 were obtained from the ATNF (Australia Telescope National Facility) website. Table 1 also includes 12 special pulsars with peak frequencies around 100 MHz. Bilous et al. (2016) and Murphy et al. (2017) classified these 12 special MPS pulsars as break-up spectral pulsars, which have radiation spectra that can be fitted by two different spectral indices and typically exhibit a breakup around GHz, with the two spectral indices positively correlated. Pulsars with such special spectra are called break-up spectral pulsars, which are distinct from those with low-frequency turnovers. These 12 special MPS pulsars have two spectral indices that are negatively correlated, which does not conform to the definition of a breakup spectrum. However, we analyzed the dispersion measure, age, and spin period of these 12 MPS pulsars and found that they completely match the evolutionary characteristics of MPS pulsars, such as small dispersion measure and large age. Therefore, we first classified these 12 special pulsars as MPS pulsars. Table 1 also includes 4-millisecond pulsars, for which there are very few samples of pulsars with a low-frequency turnover spectrum. This paper will study the peak frequency, spectral index distribution, dispersion measure distribution, pulsar period, pulsar magnetic field, and other parameters of these MPS pulsars to further discuss their statistical properties. Since these parameters of pulsars, such as spectral index and peak frequency, are affected by factors such as telescope hardware, observation time, and available bandwidth, there are certain discrepancies between the data presented in this paper and those published by other authors.

Table 1 MPS pulsars and their parameters.

Pulsar	DM/pc cm^{-3}	T_c/kyr	α	v_c/GHz	P/s	P1	B/G	Ref.
J1543+0929	35.0	27400	-1.2	0.08	0.74844	4.32×10^{-16}	5.76×10^{11}	[2]
J0304+1932	15.7	17000	-1.58	0.102	1.38758	1.30×10^{-15}	1.36×10^{12}	[4]
J0659+1414	13.9	111	-2.37	0.15	0.38493	5.49×10^{-14}	4.65×10^{12}	[4,27]
J0835-4510	67.8	11.3	-3.58	0.38	0.08933	1.25×10^{-13}	3.38×10^{12}	[4]
J1825-0935	19.4	233	-1.73	0.08	0.76902	5.24×10^{-14}	6.42×10^{12}	[4]
J1921+2153	12.4	15700	-2.41	0.055	1.33730	1.35×10^{-15}	1.36×10^{12}	[4]

J2022+2854	24.6	2870	-1.67	0.16	0.34340	1.89×10^{-15}	8.16×10^{11}	[4]
J2149+6329	129.7	35800	-2.0	0.25	0.38014	1.68×10^{-16}	2.56×10^{11}	[4,28]
J2219+4754	43.5	3090	-2.42	0.065	0.53847	2.77×10^{-15}	1.23×10^{12}	[4]
J2257+5909	151.1	1010	-2.28	0.20	0.36825	5.75×10^{-15}	1.47×10^{12}	[4]
J2305+3100	49.6	8630	-2.3	0.1	1.57589	2.89×10^{-15}	2.16×10^{12}	[4,29]
J0141+6009	34.9	49500	-1.6	0.1	1.22295	3.91×10^{-16}	7×10^{11}	[6]
J0323+3944	26	75600	-2.5	0.06	3.03207	6.36×10^{-16}	1.4×10^{12}	[6]
J0332+5434	27	5530	-3.8	0.27	0.71452	2.05×10^{-15}	1.22×10^{12}	[6]
J0358+5413	57	564	-1.3	0.1	0.15638	4.39×10^{-15}	8.39×10^{11}	[6]
J0452-1759	40	1510	-2.6	0.1	0.54894	5.75×10^{-15}	1.8×10^{12}	[6]
J0528+2200	51	1480	-3.2	≤ 0.08	3.74554	4.01×10^{-14}	1.24×10^{13}	[6]
J0543+2329	77.7	253	-1.2	0.2	0.24597	1.54×10^{-14}	1.97×10^{12}	[6]
J0614+2229	97	89	-3.8	0.08	0.33496	5.94×10^{-14}	4.52×10^{12}	[6]
J0742-2822	74	157	-1.8	0.25	0.16676	1.68×10^{-14}	1.69×10^{12}	[6]
J0814+7429	5.8	122000	-1.7	0.05	1.29224	1.68×10^{-16}	4.72×10^{11}	[6]
J0820-1350	41	9320	-2.3	0.15	1.23813	2.11×10^{-15}	1.63×10^{12}	[6]
J0826+2637	19.5	4920	-1.8	0.06	0.53066	1.71×10^{-15}	9.64×10^{11}	[6]
J1136+1551	4.8	5040	-2.0	0.03	1.18791	3.73×10^{-15}	2.13×10^{12}	[6]
J1239+2453	9	22800	-2.2	0.04	1.38245	9.60×10^{-16}	1.17×10^{12}	[6]
J1456-6843	8.6	42200	-2.6	0.15	0.26338	9.90×10^{-17}	1.63×10^{11}	[6]
J1509+5531	19.6	2340	-1.9	0.08	0.73968	5.00×10^{-15}	1.95×10^{12}	[6]
J1607-0032	10.7	21800	-1.5	0.09	0.42182	3.06×10^{-16}	3.64×10^{11}	[6]
J1614+0737	21	8100	-3.8	0.1	1.20680	2.36×10^{-15}	1.71×10^{12}	[6]
J1645-0317	36	3450	-2.4	0.08	0.38769	1.78×10^{-15}	8.41×10^{11}	[6]
J1745-3040	88.4	546	-1.7	≤ 0.3	0.36743	1.07×10^{-14}	2×10^{12}	[6]
J1752-2806	50.4	1100	-2.6	0.16	0.56256	8.13×10^{-15}	2.16×10^{12}	[6]
J1820-0427	84	1500	-2.4	0.16	0.59808	6.33×10^{-15}	1.97×10^{12}	[6]
J1829-1751	216.8	877	-1.6	≤ 0.4	0.30713	5.55×10^{-15}	1.32×10^{12}	[6]
J1844+1454	41.5	3180	-1.9	≤ 0.1	0.37546	1.87×10^{-15}	8.48×10^{11}	[6]
J1848-0123	159.1	1990	-1.9	0.3	0.65943	5.25×10^{-15}	1.88×10^{12}	[6]
J1935+1616	159	947	-4.7	0.3	0.35874	6.00×10^{-15}	1.48×10^{12}	[6]
J1946+1805	16	290000	-3.3	0.1	0.44062	2.41×10^{-17}	1.04×10^{11}	[6]
J1948+3540	129.4	1610	-2.4	0.3	0.71731	7.06×10^{-15}	2.28×10^{12}	[6]
J2018+2839	14	59700	-2.3	0.15	0.55795	1.48×10^{-16}	2.91×10^{11}	[6]
J2022+5154	22.5	2740	-2.1	0.4	0.52920	3.06×10^{-15}	1.29×10^{12}	[6]
J2046+1540	39.8	98900	-1.3	≤ 0.4	1.13829	1.82×10^{-16}	4.61×10^{11}	[6]
J2113+4644	141	22500	-2.3	0.1	1.01468	7.15×10^{-16}	8.62×10^{11}	[6]
J2157+4017	71.1	7040	-2.2	0.15	1.52527	3.43×10^{-15}	2.32×10^{12}	[6]
J2225+6535	36	1120	-3.9	0.15	0.68254	9.66×10^{-15}	2.6×10^{12}	[6]
J2321+6024	94.6	5080	-1.8	0.2	2.25649	7.04×10^{-15}	4.03×10^{12}	[6]
J2354+6155	94.7	920	-1.4	≤ 0.4	0.94478	1.63×10^{-14}	3.97×10^{12}	[6]
J1012+5307 (Millisecond pulsar)	9	4860000	-1.7	0.1	0.00526	1.71×10^{-20}	3.04×10^{8}	[11]

J1939+2134 (Millisecond pulsar)	71	235000	-2.59	0.074	0.00156	1.05×10^{-19}	4.09×10^{8}	[12]
J2145-0750 (Millisecond pulsar)	9	8540000	-2.60	0.40	0.01605	2.98×10^{-20}	7×10^{8}	[12]
J0946+0951*	15.3	4980	-2.6	0.08	1.09771	3.49×10^{-15}	1.98×10^{12}	[8]
J1115+5030*	9.2	10500	-2.2	0.13	1.65644	2.49×10^{-15}	2.06×10^{12}	[8]
J1321+8323*	13.3	18700	-2.5	0.23	0.67004	5.66×10^{-16}	6.23×10^{11}	[8]
J1532+2745*	14.7	22900	-1.6	0.15	1.12484	7.80×10^{-16}	9.48×10^{11}	[8]
J1635+2418*	24.3	65100	-2.3	0.17	0.49051	1.19×10^{-16}	2.45×10^{11}	[8]
J1741+2758*	29.1	11700	-2.0	0.13	1.36074	1.84×10^{-15}	1.6×10^{12}	[8]
J1840+5640*	26.8	17500	-1.6	0.04	1.65286	1.49×10^{-15}	1.59×10^{12}	[8]
J1907+4002*	31.0	36200	-2.1	0.27	1.23576	5.41×10^{-16}	8.27×10^{11}	[8]
J2139+2242*	44.2	12100	-0.6	0.17	1.08351	1.42×10^{-15}	1.26×10^{12}	[8]
J2308+5547*	46.5	37700	-2.0	0.20	0.47507	1.99×10^{-16}	3.12×10^{11}	[8]
J2317+2149*	20.9	21900	-2.2	0.19	1.44465	1.05×10^{-15}	1.24×10^{12}	[8]
J1543-0620*	18	12800	-1.7	0.13	0.70906	8.80×10^{-16}	7.99×10^{11}	[9]
J0809-4753	228.3	2820	-2.37	0.123	0.54720	3.08×10^{-15}	1.31×10^{12}	[5]
J0837+0610	13	2970	-3.4	0.053	1.27377	6.80×10^{-15}	2.98×10^{12}	[5]
J0922+0638	27	497	-1.62	0.041	0.43063	1.37×10^{-14}	2.46×10^{12}	[5]
J0934-5249	100	4920	-3.6	0.36	1.44478	4.65×10^{-15}	2.62×10^{12}	[5]
J0942-5657	159.7	323	-2.7	0.10	0.80816	3.96×10^{-14}	5.73×10^{12}	[5]
J0943+1631	20.3	189000	-1.6	0.059	1.08742	9.11×10^{-17}	3.18×10^{11}	[5]
J1001-5507	130.3	441	-1.75	0.19	1.43663	5.16×10^{-14}	8.71×10^{12}	[5]
J1057-5226	30	535	-2.04	0.05	0.19711	5.84×10^{-15}	1.09×10^{12}	[5]
J1651-4246	482	620	-2.19	0.09	0.84408	4.75×10^{-15}	2.03×10^{12}	[5]
J1836-1008	317	756	-2.9	0.39	0.56271	1.18×10^{-14}	2.61×10^{12}	[5]
J1913-0440	89.4	3220	-2.1	0.16	0.82594	4.07×10^{-15}	1.85×10^{12}	[5]
J0318+0253 (Millisecond pulsar)	26	4670000	-1.9	0.35	0.00519	1.76×10^{-20}	3.06×10^{8}	[18]
J0034-0721	11	36600	-3	0.055	0.94295	4.08×10^{-16}	6.28×10^{11}	[13]
J0437-4715 (Millisecond pulsar)	2.6	1590000	-2.4	0.285	0.00575	5.73×10^{-20}	5.81×10^{8}	[13]
J0630-1134	34	2770	-2.5	0.098	1.24442	7.12×10^{-15}	3.01×10^{12}	[13]
J0953+0755	3	17500	-2.6	0.104	0.25307	2.30×10^{-16}	2.44×10^{11}	[13]
J1453-6413	71	1040	-2.5	0.14	0.17949	2.74×10^{-15}	7.1×10^{11}	[13]
J1709-1640	25	1640	-1.7	0.07	0.65305	6.31×10^{-15}	2.05×10^{12}	[13]
J1932+1059	3.2	3100	-1.5	0.09	0.22652	1.16×10^{-15}	5.18×10^{11}	[13]
J2048-1616	11.5	2840	-1.1	0.18	1.96157	1.10×10^{-14}	4.69×10^{12}	[13]

3.1. Low Frequency Inversion Spectrum Pulsar Peak Frequency Statistics

The distribution of peak inverse frequency (PIF) for MSP pulsars is shown in Figure 1. Among the 82 MSP pulsars selected from the literature, all of their PIFs fall within the range of 0-0.45 GHz. The average PIF of the 76 known MSP pulsars is 0.15 GHz, while the average PIF of the four-millisecond pulsars is 0.24 GHz, which is higher than that of the 76 MSP pulsars. It can be clearly seen from the figure that 53 MSP pulsars have PIFs between 0.05 GHz and 0.2 GHz, accounting for 70% of the total, indicating that most MSP pulsars experience PIF reversal within this range. 19 MSP pulsars have higher PIFs ranging from 0.2 GHz to 0.45 GHz, with an average PIF of 0.29 GHz, accounting for 25% of the total. Among them, two MSP pulsars have the highest PIFs, reaching 0.4 GHz, including the millisecond pulsar J2145-0750. Kondratiev et al. (2017) studied the radio emission spectrum of millisecond pulsars such as J1939+2134 and J2145-0750 and measured their flux densities in the frequency range of 110 MHz to 180 MHz. Through model fitting, they found that J1939+2134 did not reach its peak frequency at 74 MHz as proposed by Kuniyoshi et al. (2015), and no PIF reversal was observed. They believed that the measured flux density values were inaccurate and underestimated the actual values. They also believed that the absence of PIF reversal was due to the fact that millisecond pulsars are weak radio sources, and compared with ordinary pulsars, they are more affected by free-free absorption in the interstellar medium at low frequencies. Wang et al. (2021) discovered J0318+0253, the weakest MSP radio source ever observed, using the FAST (Five-hundred-meter Aperture Spherical radio Telescope), and confirmed it as an isolated millisecond pulsar that has lost its companion. They determined its peak frequency be approximately 350 MHz and suggested that the PIF reversal phenomenon may be related to the environment surrounding the millisecond pulsar or the absorption of radiation by plasma in its magnetosphere.

Table 1 lists 5 MSP pulsars, which exhibit a much lower frequency of mode-changing behavior compared to other MPS pulsars. They have higher peak reversal frequencies, with J0318+0253 having a confirmed peak frequency of around 0.35 GHz, and J2145-0750 having a reversal frequency reaching up to 0.4 GHz. Additionally, it can be seen from the table that the ages of these 5 MSP pulsars are much older than those of ordinary MPS pulsars. Furthermore, J2145-0750, which has the steepest high-frequency spectrum and the oldest age among them, suggests that it is in a later evolutionary stage, with a trend towards steeper spectra.

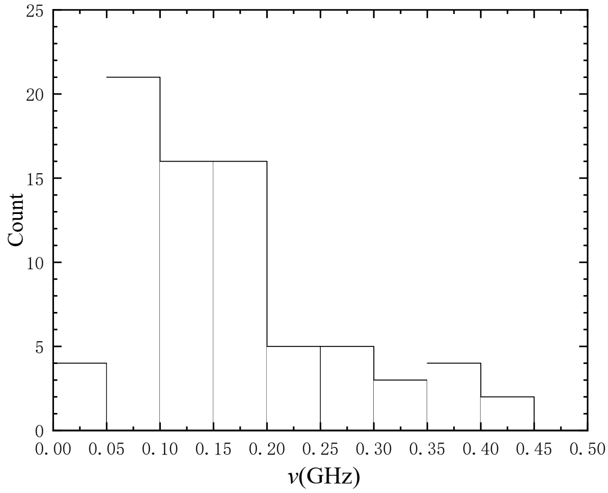

Figure 1 Peak frequency distribution of MPS pulsars.

3.2. Spectral Index Distribution

The statistical distribution of spectral indices for MPS pulsars is shown in Figure 2. The average spectral index of 82 MPS pulsars is -2.23, while that of the 4-millisecond pulsars (MSPs) is -2.24. From the average spectral index, there seems to be no difference between MPS and MSP pulsars. 74% of the 61 MPS pulsars have spectral indices in the range of [-2.5,-1], while 12% of the 10 MPS pulsars have spectral indices in the range of [-5,-3], with an average spectral index of -3.54, indicating a steeper spectrum. Among them, J2225+6535 has the steepest spectral index in the sample. Lorimer et al. (1995), Malofeev et al. (1996), Kijak et al. (1998), and Maron et al. (2000) have expanded the sample of pulsar radiation spectra and determined spectral indices of -1.6, -1.7, -1.9, and -1.8, respectively. Jankowski et al. (2018) studied the radio emission spectra of 441 pulsars and determined the average spectral index of pulsars to be -1.6, including 10 MPS pulsars with low-frequency turnovers determined using the free-free absorption model, with an average spectral index of -2.29, accounting for 2.3% of the sample.

Overall, the spectral index of millisecond pulsars (MPS) is smaller than that of other pulsars, indicating that the spectral shape of MPS is steeper. Bates et al. (2013) analyzed the spectral index distribution of a large number of pulsars using population synthesis techniques and likelihood analysis and determined a spectral index of -1.4. They suggested that pulsars with flatter spectra are easier to detect at frequencies above 2 GHz, while those with steeper spectra are easier to detect at frequencies below 1 GHz. This once again confirms the conclusion that the average spectral index of MPS is smaller than that of other pulsars, and that their spectral shapes are steeper. Bates et al. (2013) also noted that when studying the correlation between the spectral index and other parameters (such as magnetic field and age), the frequency selection of pulsars must be taken into account. Kondratiev et al. (2019) studied a large sample of millisecond pulsars and found that at least one-third of them have uncertain spectral indices, which directly affect their flux density measurements. This suggests that the flux density of millisecond pulsars may be related to their spectral indices.

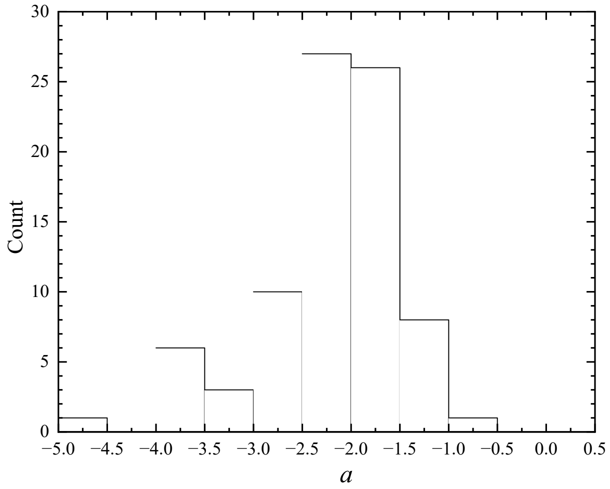

Figure 2 Spectral index distribution of MPS pulsars.

3.3. Dispersion Measure Distribution

Before pulsar signals reach Earth, they undergo dispersion due to the scattering effect of free electrons on electromagnetic waves. This results in the arrival time of pulse signals of different frequencies being different, with high frequencies arriving first and low frequencies arriving later. This phenomenon is called pulsar dispersion. van Straten (2011) used a digital signal processing software applied to radio pulsar astronomy to study dispersion measures and found that the dispersion measure is proportional to the product of the electron density and the distance along the path of the pulsar radio wave, indicating that the dispersion measure can be used to determine the electron density along the path of the pulsar signal. Thus, the larger the dispersion measure, the higher the density of the interstellar medium surrounding the pulsar. We provide a statistical distribution of dispersion measures for 82 MPS pulsars in Figure 3, where the average dispersion measure for 82 MPS pulsars is 59.1 $\mathrm{pc\,cm^{-3}}$, and the average dispersion measure for 5-millisecond pulsars is 23.5 $\mathrm{pc\,cm^{-3}}$. This suggests that the dispersion measure of millisecond pulsars with inverted spectra is generally lower than that of MPS pulsars. Among the 56 MPS pulsars, 68% have a dispersion measure ranging from 0-50 $\mathrm{pc\,cm^{-3}}$, while 29% have a dispersion measure ranging from 50-250 $\mathrm{pc\,cm^{-3}}$, and 2 MPS pulsars even have a dispersion measure exceeding 300 $\mathrm{pc\,cm^{-3}}$, indicating a higher density of the interstellar medium surrounding these two pulsars. For comparison, we also analyzed the dispersion measure of GPS pulsars, which have an average dispersion measure of 333 $\mathrm{pc\,cm^{-3}}$, much higher than that of MPS pulsars. The magnetar J1745-2900 with a GPS spectrum has a dispersion measure as high as 1778 $\mathrm{pc\,cm^{-3}}$.

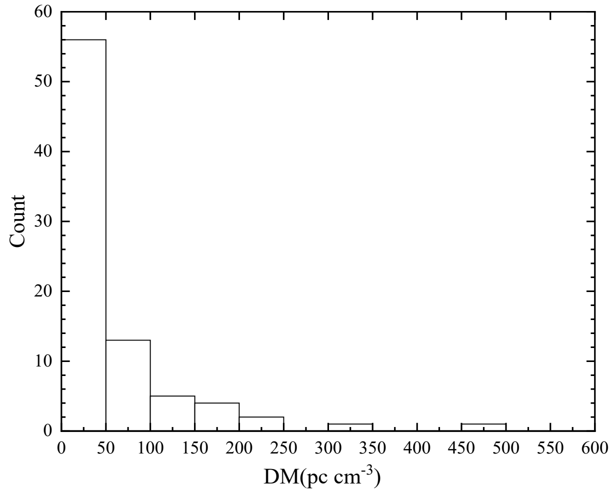

Figure 3 Dispersion measure distribution of GPS pulsars.

We also investigated the correlation between the dispersion measure (DM) and the period of MSP pulsars. We employed the Pearson product-moment correlation coefficient, a linear correlation coefficient used in statistics to measure the correlation between two variables. Its value ranges from -1 to 1, and the absolute value of the coefficient indicates the strength of the correlation. Figure 4 shows the correlation between DM and the period of 77 MPS pulsars excluding MSPs, with a correlation coefficient of -0.11, indicating a weak negative correlation. From Figure 4, we also observed that the variation in DM does not strongly affect the period. The DM is related to the interstellar medium density, and the increase in the interstellar medium density around the pulsar did not affect its rotation period. At low frequencies, the interstellar medium undergoes more severe free-free absorption, which causes the pulsar to undergo peak inversion. Furthermore, we noticed that J0835-4510 and J0528+2200 deviate significantly from the linear correlation in Figure 4, which may be caused by the variability of DM.

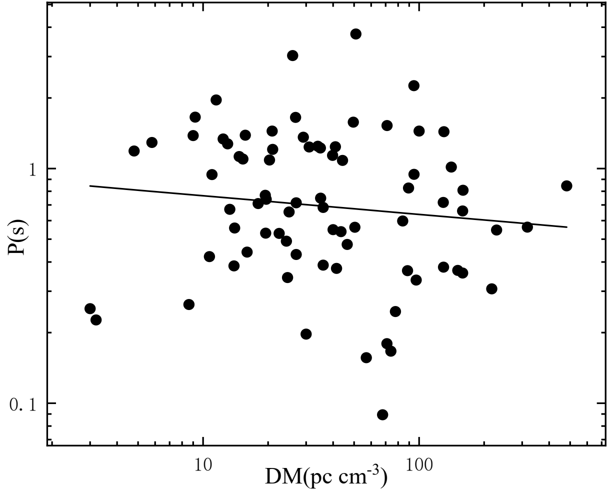

Figure 4 Correlation between the periods and dispersion measure of MPS pulsars.

3.4. Pulsar Surface Magnetic Field

The distribution of magnetic field strengths for 82 MSPs is shown in Figure 5, with 62% of the MSPs having magnetic field strengths between 1.09×10^{12} G and 8.71×10^{12} G. Among these MSPs, 5 of them have much weaker magnetic fields, with an average magnetic field strength of 4.6×10^{8} G. This is because that millisecond pulsars have much older ages, with an average age of 4×10^{6} ky, compared to other pulsars, and their magnetic energy is depleted over time, resulting in weaker magnetic fields. We also compared the magnetic field strengths of GPS pulsars and found that their average magnetic field strength is 2.2×10^{13} G, which is higher than that of MPS pulsars. This is because GPS pulsars are generally younger, with ages much smaller than those of MPS pulsars, and their magnetic energy is consumed more slowly over time, resulting in stronger magnetic fields. We also analyzed the correlation between the age and magnetic field strength of MPS pulsars, as shown in Figure 6, obtaining a strong correlation coefficient of -0.79. This indicates that the magnetic field strength of MPS pulsars decreases rapidly with increasing age.

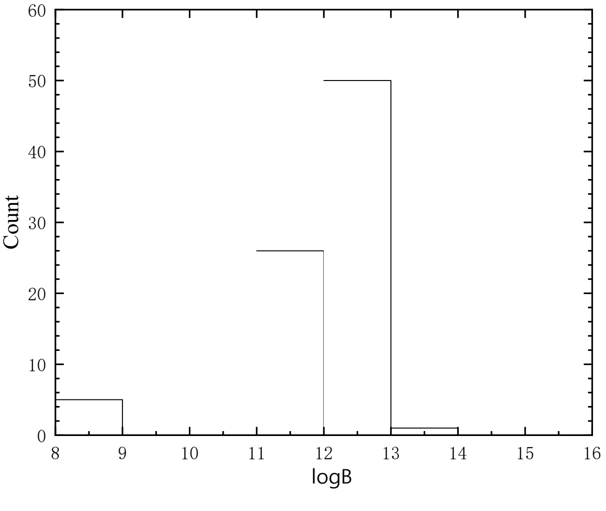

Figure 5 Magnetic field distribution of MPS pulsars.

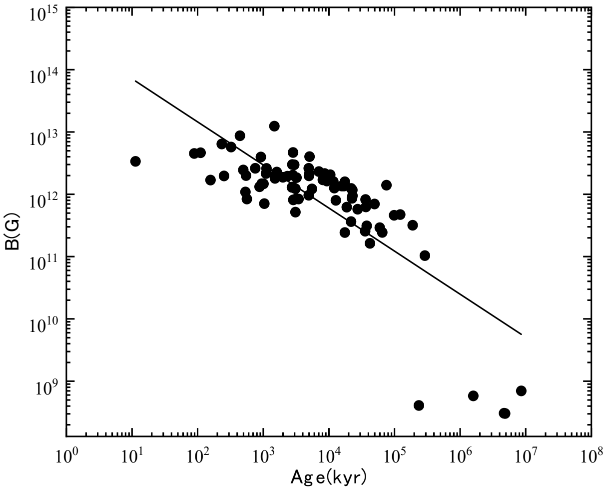

Figure 6 Correlation between magnetic field and age of MPS pulsars.

4. Discussion

A total of 82-millisecond pulsars (MPS) were surveyed in this study, with MPS pulsars defined as those with peak frequencies below 0.45 GHz in their inverted spectra. Another type of special spectrum is the gigahertz-peaked spectrum (GPS), which typically has its peak frequency near the GHz range. However, further investigation is required to explore the differences in statistical parameters between these special spectra and the MPS spectra.

As time passes, the radiation intensity of pulsars also undergoes slow variations, which may be related to free-free absorption in the interstellar medium or intrinsic changes within the pulsars themselves. Therefore, the spectral index of pulsars needs to be measured multiple times to obtain the average spectral index of the total standard pulse profile. However, most pulsar signals are very weak, making high demands on observation equipment. Furthermore, interference from interstellar medium density and scattering can also affect the distance between pulsars and observers, making it difficult to ensure the accuracy of the average spectral index. Therefore, there are certain errors in the spectral index and peak frequency presented in Table 1 of this study, which have a significant impact on the analysis results.

The formation of inverted spectra is typically associated with free-free absorption in the interstellar medium, suggesting that the radiation mechanism of MPS pulsars is likely free-free absorption. Investigating the low-frequency inversion phenomenon of millisecond pulsars is useful for improving pulsar timing accuracy, exploring the material composition within millisecond pulsars with binary systems, and detecting low-frequency gravitational wave signals to understand the processes of black holes and early universe evolution. Therefore, the low-frequency inversion phenomenon of millisecond pulsars is of great interest to researchers. In recent years, Kuniyoshi et al. (2015), Wang et al. (2020), Kaur et al. (2019), and Lee et al. (2022) have conducted research on the inverted spectra of millisecond pulsars. Kondratiev et al. (2016) studied 48-millisecond pulsars in the low-frequency observation range of 110 MHz-188 MHz, analyzing the pulse profile, pulse width, and flux density, and comparing them with the values obtained at high frequencies. They proposed the viewpoint that at least one-third of millisecond pulsars have flux densities lower than the measured values and obtained the pulsar radiation signal by accumulating the profile, discussing the relationship between dispersion measure and pulsar profile. They found that 35% of millisecond pulsars have very narrow pulse profiles, 25% have relatively wide pulse profiles, and 40% have unclear pulse profiles, suggesting that this may be related to scattering, including J1939+2134, a millisecond pulsar associated with scattering. They believed that the low-frequency inversion phenomenon of millisecond pulsars occurs in the narrowest pulse profile pulsars.

This article also analyzes the correlation between the peak inverse frequency of MPS pulsars and their characteristic age. Among the 71 known MPS pulsars with peak frequency data, excluding the five MSP pulsars, the correlation coefficient is only -0.22, indicating a weak negative correlation between age and peak frequency. Two pulsars, J1136+1551 and J0922+0638, deviate significantly from the correlation line. Excluding these two pulsars from the analysis, the correlation coefficient between age and peak frequency for the remaining 69 pulsars is -0.27, indicating a slightly stronger negative correlation, but still a weak one. Therefore, the characteristic age of MPS pulsars has little influence on peak frequency.

Compared to GPS pulsars, MPS pulsars are generally older. Among them, 33 MPS pulsars are older than 1×10^4 kyr, and the average age of the five MSP pulsars is much younger than that of MPS pulsars, indicating that they are likely to be dead pulsars. J2145-0750 is the oldest MSP pulsar in the sample, with the smallest spectral index, indicating a steep spectral index in its late evolution. J1935+1616 has the smallest spectral index among the MPS pulsars in the sample, with a spectral index of -4.7, and a relatively large dispersion measure compared to other pulsars, indicating that this pulsar is in its early evolution stage with a high interstellar medium density. This article also analyzes the correlation between the dispersion measure and period, and between the dispersion measure and inverse frequency, with correlation coefficients of -0.11 and 0.34, respectively. The weak correlation between the dispersion measure and period, and between dispersion measure and inverse frequency of MPS pulsars, combined with the previous analysis, further indicates that the interstellar medium may not be a significant factor in MPS pulsar evolution. We also analyzed the correlation between the dispersion measure and period, and between the dispersion measure and inverse frequency of GPS pulsars, with correlation coefficients of 0.59 and 0.47, respectively, indicating that the interstellar medium has a significant impact on GPS pulsar evolution.

The low-frequency inversion phenomenon of pulsars is very advantageous for studying the interstellar medium around pulsars and improving the timing accuracy of pulsars. However, improving timing accuracy requires minimizing the impact of the interstellar medium, which can cause pulse profile distortion, affecting the accuracy of the dispersion measure. Currently, Kaur et al. (2019) have developed a beam-forming software that successfully measures the accurate value of DM variation for pulsar J2241-5236. As people continue to explore pulsars, the accuracy of DM measurements and the sensitivity to DM variations can be further improved in the future.

5. Conclusion

Low-frequency turnover spectra are a type of special spectra with a peak frequency of around 100 MHz. Pulsars with such special spectra are called low-frequency turnover spectrum pulsars, and in this paper, we introduce the name MPS pulsars based on the definition of low-frequency turnover spectra. Low frequency greatly affects the free-free absorption of the interstellar medium, so the radiation mechanism of this type of special spectrum is likely to be caused by the free-free absorption of the interstellar medium. In this paper, we collected various parameters of 82 MPS pulsars and analyzed their dispersion measure distribution, spectral index distribution, and turnover frequency distribution. We found that most MPS pulsars have small DM values, distributed between 0-50 $\mathrm{pc\,cm^{-3}}$, with small average dispersion measure, and spectral indices mostly distributed in the range of [-2.5, -1], with average spectral indices smaller than those of other pulsars. The spectra of MPS pulsars are steeper than those of other pulsars, and their turnover frequencies mostly range between 0.05 GHz to 0.2 GHz. We also analyzed the correlation between the dispersion measure and period of MPS pulsars and found that their correlation is weak, indicating that the interstellar medium may not affect the evolution of MPS pulsars. Finally, we analyzed the correlation between the magnetic field and the age of MPS pulsars, and found a strong negative correlation, indicating that the magnetic energy of MPS pulsars decreases with age.

Acknowledgements

Study on the Method of Establishing Atomic-Pulsar Time Scale(number: 11873049)

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